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Phenomena Occurring at Explosive Metal Interfaces

(Strain-Rate Dependency in Metals)

by

D. Braslau

(Physics International Company)

SEPTEMBER 1966

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AIR FORCE ARMAMENT LABORATORY

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FOREWORD

This report was prepared by Physics International Company, San Leandro, California, under Air Force Contract No. AF 08(635)-4865, Project No. 2508, Task No. 2508-01. The work was administered under the direction of the Air Force Armament Laboratory, AFSC, Eglin Air Force Base, Florida. Technical monitoring of the contract was performed by Mr. Lenton Hill.

This study was begun in December 1964 and concluded in April 1966. The final report was submitted in April 1966. The contract effort was performed by personnel of Physical International Company. Project supervisor was Dr. David Bernstein, Head of Experimental Hydrodynamics. Project Manager was Paul Hertelendy.

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ABSTRACT

A thorough summary of the literature treating strain-rate behavior in metals, 1868-1965, is given. Attempts to apply these theories and results to specific problems to be solved on a computer are discussed. A proposed qualitative description of the behavior of metals under dynamic loading is also presented.

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SECTION I

INTRODUCTION

The dynamic or sudden application of loads in general enables materials to withstand higher stresses before yielding than would be the case with loads statically applied. Therefore, in employing the concept of a yield criterion in dynamic problems, one uses the expression "dynamic yield stress" to indicate that the behavior of the material here is different from the static case.

The exact nature of this effect is not yet fully explained. The questions which may be posed are: Is the effect of dynamic loading constant, or does it depend upon the actual rate of loading? Is this an effect of the rate of straining or the amount of energy put into the system; in other words, how is this to be incorporated into the constitutive equations for the material?

Studies were made initially on thin wires under tensile impact. Subsequently these were extended to thin bars impacted in compression. These bars became shorter and shorter, causing the complex geometry to place the original assumptions in question. The most recent attempts at studying the rate effect have been made in uniaxial strain, i.e. the impact or explosive loading of flat plates, where the edge effects do not interfere with the experimental observations over very short times.

In this report the international research and progress in strain-rate effects of metals under dynamic loading is summarized and some modifications of proposed theories are presented. A summary of the author's numerical calculations based on the Physics International "PIPE"* and "POD"* computer codes is also included.

A survey of the literature is given in four parts:

(1) Studies of dynamic effects in thin wires, thin rods, and short rods.

(2) Studies employing plates or blocks with uniaxial strain.

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*Godfrey, C. S., Andrews, D., Teatum, E., and Trigg, M., "Calculations of Underground and Surface Explosions," PIFR-013, Physics International Company, San Leandro, California (November 1965).

(3) A very brief discussion of the literature on the microstructural changes that could produce strain-rate effects.

(4) Mathematical theories which generalize the original one-dimensional theories to more dimensions, and which incorporate more general constitutive equations for strain-rate dependent materials.

Following the literature review, some modifications and applications of the theories proposed are presented.

An initial study of impact of aluminum rods was carried out on the PIPE computer code, but numerical difficulties with the program prevented their successful conclusion. Thereafter, some modifications of proposed theories were programmed and carried out using the POD computer code. The existing computational technique developed instabilities, however, whenever strain-rate effects were introduced, and new techniques will have to be evolved in order to reduce the strain-rate propagation problem to an algorithm.

In the section entitled "Discussion and Conclusions," a proposed qualitative model for dynamic behavior is presented, along with some final remarks on possible extension of the present work.

SECTION II

SURVEY OF THE LITERATURE

A. DYNAMIC EFFECTS IN THIN WIRES, THIN RODS, AND SHORT RODS

The initial experiments on the effect of rate of loading of materials in the plastic range upon stresses, strains, and velocities were done on thin wires and rods.

P. Ludwik (1909) was one of the first investigators of the dynamic effect of loading in thin wires. He performed tests with constant force and with constant strain rate. He proposed the equation

$$R = R_o + k \sqrt[n]{v}$$

to relate the internal friction as a function of loading velocity, where R_o, k, and n are material constants. He found that a logarithmic curve fits his data rather well. He also found that with higher velocity of loading, the material can withstand more stress at a given strain.

If the logarithmic curve is described by

$$x = k a^y$$

and we set $x = v + k$ and $y = R - R_o$, this leads to the expression

$$v + k = k a^{R - R_o}$$

or

$$\dot{\epsilon} + k = k a^{\sigma - \sigma_o}$$

or

$$\dot{\epsilon} = k(a^{\sigma - \sigma_o} - 1)$$

In other words, the R and R_o, which Ludwik identifies as material constants, are directly related to the dynamic and static yield stress of the material.

Deutler (1932) summarized the experiments on stress versus deformation rather well. He discussed in some detail the models of Ludwik and Prandtl (1928) among others. He carried out some well-controlled experiments on iron and copper specimens in a dissertation under Prandtl.

Deutler based his equations on those derived by Prandtl in the form

$$\sigma_\nu - \sigma_1 = (\text{const.}) \times \left(\ln \frac{v_\nu}{v_1} \right)$$

where σ_ν is the stress related with the velocity v_ν and σ_1 is the stress related to the velocity v_1. The term $\ln$ indicates natural logarithm.

Sokolovskij (1948) proposed the following relations to describe the material in plastic wave propagation in bars:

$$E\dot{\varepsilon} = \dot{\sigma} \quad |\sigma| \leq \sigma_o$$

$$E\dot{\varepsilon} = \dot{\sigma} + \varkappa kF(|\sigma| - \sigma_o) \quad |\sigma| \geq \sigma_o$$

$$F'(z) > 0 \qquad F(0) = 0 \qquad \varkappa = \text{sign } \sigma$$

Here k is a material constant. This relation reduces in the linear case to that of Hohenemser and Prager (1932) and Ishlinskij (1940).

Clark and Wood (1949) employed a rapid loading machine to keep stress constant over time. The rise time was in milliseconds, and the resolution of the order of one millisecond. Curves were obtained of stress versus time to initiate plastic deformation, of the form

$$t = 4 \times 10^{-4} \left(\frac{\sigma - \sigma_o}{\sigma} \right)^{-6}$$

where t is in seconds.

These times are too long to be of special interest in this report. For metals other than steel, the tests showed no delay for the initiation of plastic deformation, and no significant stress rise.

von Karman and Duwez (1950) reported work that was carried on during World War II. This paper initiated the study of propagation of inelastic waves in rods in the United States and is the first of numerous articles in this field.

They proposed what is now universally known as the strain-rate independent theory of plastic wave propagation in rods. By this theory, the velocity of propagation at a stress level was directly proportional to the square root of the slope of the stress-strain diagram from a one-dimensional tension test. The equations used were truly one-dimensional, with only one independent variable of length present in the equations.

They performed tests on annealed copper wire, which they summarized:

> "It may also be pointed out that the deviation between the two curves (computed and measured) certainly reflects some influence on the strain rate, showing that the assumption of a stress-strain curve independent of the rate of strain is not entirely justified."

They discussed this deviation in general terms.

Malvern (1951), in two papers, presented a study of plastic wave propagation in rods, taking into account the effect of strain rate. His equations were based on those of Ludwik and Deutler. In order to simplify the numerical integration of the equations governing wave propagation in rods, he employed an idealized form of the stress-strain rate law, where the plastic strain rate is directly proportional to the excess stress over the stress at the same strain in a static test. His general flow law had the form:

$$E_o \dot{\epsilon} = \dot{\sigma} + g(\sigma, \epsilon)$$

For simplicity, this can be idealized to the form

$$E_o \dot{\epsilon} = \dot{\sigma} + k[\sigma - f(\epsilon)]$$

It can be noted that Deutler's form is

$$\sigma - \sigma_o = A \ln \frac{\dot{\epsilon}}{B}$$

To compare with the static case (i.e., $\dot{\epsilon} = 0$), one may write this as

$$\sigma - \sigma_o = A \ln(1 + b\dot{\epsilon})$$

Then

$$\dot{\epsilon} = \left[e^{\left(\frac{\sigma - f(\epsilon)}{A}\right)} - 1 \right] \frac{1}{b}$$

So the strain rate depends on the quantity $\sigma - f(\epsilon)$, or, more generally,

$$E_o \dot{\epsilon}^P = g(\sigma, \epsilon)$$

where ϵ^P represents the plastic part of the strain.

Malvern stated that the strain-rate effect can account for the discrepancy observed in the stress-time variation at the fixed end of impact specimens.

He felt that an exponential dependence of plastic strain rate on the excess stress would be more realistic. He wrote:

> "By a suitable choice of the function $g(\sigma, \epsilon)$ ... the stress strain curve for various strain rates can be made to approximate the elastic line for any desired range of strain above ϵ_y so that an apparent increase in yield stress is produced. This formulation implies that the material is brought to a state of incipient plastic flow after a given amount of elastic strain, independent of the elastic strain rate, but that the plastic flow requires time in which to become appreciable so that the additional strain beyond ϵ_y is at first mainly elastic."

Sternglass and Stuart (1953) carried out experiments on flat strips of cold-rolled copper. They drew a large number of conclusions, among them the following: (a) the velocity of propagation of the wave front is that of the elastic wave; (b) the velocity of propagation of any part of the pulse is much greater than that given by the tangent modulus at the point of the stress-strain curve to which the material has been pre-stressed; (c) there is notable dispersion; (d) the amplitude of a plastic strain does not decay linearly with distance; and (e) the rate of decrease of amplitude with distance appears to be an increasing function of the amplitude of the strain at the point of impact.

Riparbelli (1953) discussed further the concept of time lag in plastic deformation. He concluded that the velocity of propagation of the front is elastic, and that the wave damps out, while flattening and spreading. "A tail, or wake, of residual deformation is observed after the wave has passed through the station, as well as an increment of residual elongation of the bar caused by the impact." He concluded that an excess of stress is reached behind the front, and then flow begins to occur in time. He employed the relation

$$\dot{\epsilon}^{P} = k(\sigma-\sigma_{st})$$

which determines just how fast the material flows under a given state of stress. The term σ_{st} indicates static yield stress.

In 1954 Riparbelli continued experiments to check the proposals of his previous papers. He restated the problem in his paper entitled "A Paradox in the Theory of Impact" (1954).

Alter and Curtis (1956) studied the effects of strain rate in lead bars and concluded, "In contrast to the rate independent theory, the strain-rate model predicts correctly that the two pulses produced by the double pressure step should coalesce and that the strains near the mean values of the first and second pulses should, respectively, travel slower and faster than the same strains of the larger single pulse." They also discussed the deviation from linearity of the effects of strain rate.

Vigness, Krafft, and Smith (1957) studied yield stress versus delay time to yield, and examined the effect of preloading at one instant of time on the delay time phenomenon at a later instant.

Bell (1959-60) used a diffraction grating technique he developed to measure strain-time and surface angle-time histories in impact experiments of annealed aluminum. He concluded that the strain-rate independent theory of plastic wave propagation was verified.

His stress strain curves for dynamic and static experiments coincided for this material.

Turnbow (1959) gave a rather thorough summary of some of the experiments to that date and found that strain rate increased the ultimate stress for copper and aluminum alloy.

Ripperger (1960) defined yield stress where the slope of the stress-time curves changes "drastically." For many materials, this definition is woefully inadequate. Interestingly enough, he found that the "yield stress" decreased with increase in length of the specimen for copper. These curves are similar to those derived by Ivanov et al (1963) for steel. He found that copper is strain-rate sensitive and lead is not, and concluded that the lateral inertia effects were small.

Steidel and Makerov (1960) carried out tensile tests of specimens at moderate rates of strain such as 100 sec^{-1}. Even here they obtained a slight rate dependence.

Karnes (1960) performed experiments on copper and lead bars, using a Hopkinson pressure bar to measure stresses in the specimens. He compared these results with computed values, using strain-rate and nonstrain theories. He felt that the exponential strain-rate dependence would give reasonable results.

Tapley (1960) carried out a mathematical study of plastic wave propagation in copper rods, taking into account the lateral inertia and shear. A linear strain-rate dependence after Malvern was also incorporated. Radial displacements were assumed to vary linearly in the radial direction, and it was assumed that the longitudinal stress was much larger than the lateral stress. This led to simplifications. He found that the lateral inertia model gave larger oscillations than predicted by the simpler theory.

Kolsky and Douch (1962) performed tests on short bars (fired from air guns) of pure copper and aluminum, and aluminum alloy. They found no appreciable strain-rate dependence for the alloy.

Bell (1963) tested materials at different temperatures employing his diffraction grating technique for measurements. He found that all materials tested were strain-rate independent. He questioned in his paper the advisability of using strain gages, indicating that there is a time lag in their response.

Chiddister and Malvern (1963) performed tests on metals using a split Hopkinson pressure-bar. They found that the strain-rate dependence could be fitted with a power function (log-log plot) or by a semilog plot, but that the power function was better for lower strain rates.

Rajnak and Hauser (1963) used thin-walled tubes to eliminate "breathing" in aluminum, in a thin-wafer technique. They obtained some initial peaks of stress. They concluded that predictions of plastic strains based on the quasi-viscous behavior of dynamically impacted materials agree very well with experimental observations if the stress, strain, and strain-rate behavior of the material are known.

Marsh and Campbell (1963) performed tests on mild steel specimens and found that the linear strain-rate law is insufficient to describe the results. They proposed a fractional power law.

Davies and Hunter (1963) dynamically tested inorganic and organic solids by means of the split Hopkinson pressure-bar. They determined dynamic yield stresses for metals of an order of 3.5 times the static yield stress.

Smith (1963) tension-tested metals with strain rates up to 200 sec^{-1}. Even at this comparatively low rate, there was a pronounced rate effect.

Karnes (1963) used X-cut quartz crystals and strain gages simultaneously on cold worked and pure aluminum to measure stresses under impact. Stress and strain vs time curves were determined from the tests. There were significant oscillations in the stress curves, and a pronounced peak at the beginning. The peak is under one microsecond in width. Karnes could not conclude whether this is associated with the thickness of the quartz crystal or with reflections from the free surfaces of the rod. Definite strain-rate effects were noted, although the amount of cold working did not seem to increase the rate effect. It was decided that lateral inertia played a minor role in the stress level measured. He concluded that neither the linear nor the exponential strain-rate law as formulated by Malvern is adequate in describing the behavior of high-purity aluminum for strain rates up to 4000 per second.

Grimm (1964) examined the propagation of waves in a lead bar. He used circumferentially wound strain gages and found that a lateral inertia model for nonlinear strain propagation predicted some of the observed results.

Efron (1964) carried out experiments on aluminum rods under impact. Two series of tests were performed, one with electromagnetic transducers and the second with etched foil resistance strain gages.

It was interesting to note that the transient strain records showed consistently lower propagation speeds than those based on velocity records. This indicated that the strain gage response actually lags behind the arrival of the front. Efron stated that although it lagged behind the strain in the material, it was quite feasible that this was a real effect of time delay in the material. He pointed out Bell's suggestion in 1960 that the strain-rate dependence indicated from earlier wave propagation experiments had been due to a lag in the gage response. He found that his computer solution based on characteristics and a linear rate dependence were consistent with a single dynamic curve.

De Vault (1964) solved the problem of wave propagation in a rod, including lateral inertia effects, by means of finite difference equations. He concluded that some observations set forth as proof of the existence of a strain-rate effect might equally be explained, at least qualitatively, on the basis of a lateral inertia effect.

Ting (1964) studied the general properties of solutions of the problem of impact of a visco-plastic rod.

Ting and Symonds (1964) solved analytically the problem of impact of viscoplastic rods, using a linear stress-strain rate law.

Symonds and Ting (1964) used a power law relation between strain rate and dynamic overstress and derived more analytical curves relating to the problem.

Lubliner (1964) generalized the theory of strain-rate dependent plastic wave propagation in bars, based on a general quasi-linear constitutive equation.

Malvern (1965) expressed doubt about the validity of using strain gages for dynamic measurements. He commented on Bell's proposal that a small field is set up by moving dislocations, which adds or detracts to the strain gage reading. Malvern had found that the strain-gage was in error up to 25 percent at strain rates of 300 sec^{-1}.

He proposed therefore that other means of measurement be used, either of particle velocity or strain at the surface. He evidently favored a diffraction or photographic technique, and felt that the average stress in a rod is a poor indicator of material behavior . He agreed that one needs to have more information on the internal behavior of the rod and suggested transparent specimens, but unfortunately most transparent materials are highly viscoelastic.

B. STUDIES EMPLOYING PLATES OR BLOCKS WITH UNIAXIAL STRAIN

The complexity of the state of stress in rods and short bars has suggested an alternative approach taking advantage of an increased technological ability to produce controlled experiments with flying plates and high explosives. This is a very recent trend, and as yet only a few papers have been published giving results bearing specifically on strain-rate effects.

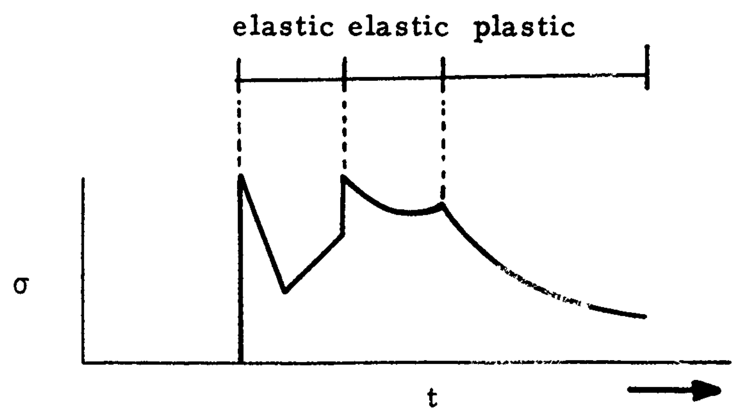
Minshall (1955) developed an experimental technique to study elastic-plastic phenomena in explosively loaded metals. He used pin contactors and tourmaline crystals. The separation of precursor and plastic wave was quite distinct. Dynamic yield points were determined for steels and tungsten. The pressure increased behind the precursor. In SAE 1040 steel the initial propagated pressure was 6 kb (87,000 psi). It then increased before the arrival of the plastic wave to 12 kb.

It was concluded that the pressure in the elastic wave does not depend upon the plastic wave pressure. He also suggested calling the "dynamic

elastic limit" the "Hugoniot elastic limit" to differentiate between the definition used here and the engineering terms "yield stress" and "ultimate strength."

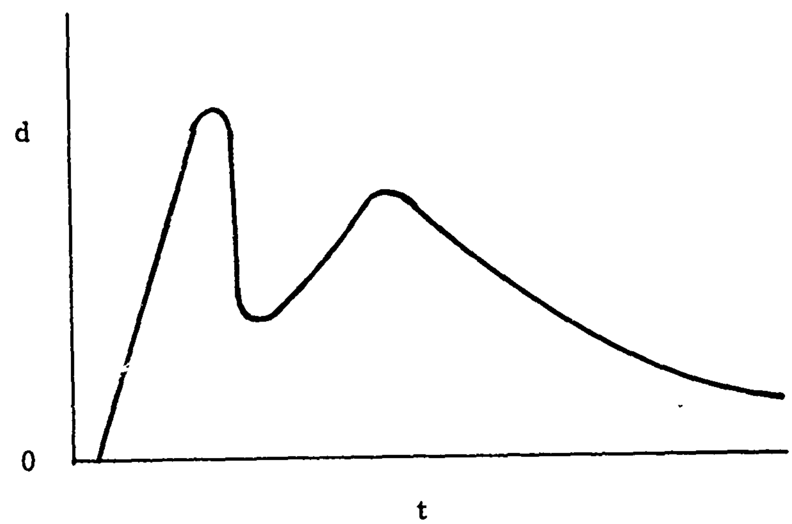
Broberg (1956) pointed out that the rigidity is affected by the pressure, a factor which has not been widely discussed, and noted that quantitative results were lacking.

His proposed curve was as follows



where the arrow indicates arrival of the plastic wave front. Broberg failed to point out that once the dynamic stress is high, yielding occurs and continues to occur, so that there is no second "elastic region." The initial peak agrees with the dynamic increase of stress, but is probably too high, because of some upper limit on the dynamic yield stress.

His experiments involved the impacting of a steel ball on the surface of a flat steel plate. The curves which he obtained are of the following shape:



where the coordinates are surface displacement and time. The experiments warrant additional examination and discussion.

Jones, Neilson, and Benedick (1962) reported explosive loadings of plates, and used X-cut quartz crystals to measure the stress pulses. For iron and steel the explosive geometry was such that the pressure remained constant over the center for about 2 - 3 microseconds, as compared with an observation time of 1 microsecond. For duralumin, a special explosive lens was developed to yield pressures to about 25 kb.

The results showed definite precursors. For the hardenable materials such as SAE 4340-RC40 and the duralumin, there was no relaxation after the precursor, but for non-hardenable materials such as the ARMCO iron and SEA 1018 steel, there was a definite relaxation observed. This would indicate that the hardening materials harden faster than they relax.

The relaxation times were on the order of 0.1 millisecond. There seemed to be an upper limit to the dynamic yield stress, for it was never so high that the second wave disappeared.

Costello (1957) carried out explosive loading of steel blocks. He obtained straight lines for plots of log of yield strength vs time to yield .

Duvall and Alverson (1962) dealt with uniaxial strain and studied the attenuation of shock waves in a stress-relaxing solid.

The plastic strain-rate was set equal to Nbv, where N is the dislocation density, b is the slip vector, and v is the velocity of the dislocations. They pointed out that W. G. Johnston had suggested that the plastic strain rate was a function of the plastic strain and shear stress alone, and not an explicit function of time. Since the correct choice of a function for plastic strain rate is difficult except for a very few cases, they decided upon a simple relaxation function

$$F(\sigma_x, \rho) = (\sigma_x - \sigma_x^o) \frac{1}{T}$$

The relaxation term is then introduced directly into the equation for the normal stress across the wavefront. Their solutions showed a decay in the precursor (they introduced an initial amplitude of 100 kb, but did not amplify this point). They compared these results with SIOOX Quartzite, although the two configurations are not identical. The approximate relaxation time was found to be 1.2 μsec. They assumed that the initial amplitude of the precursor was purely elastic and that there is no upper limit to the amplitude of the elastic precursor. This may indeed be the case for quartz.

Ivanov, Novikov, and Sinitsyn (1963) reported results for an investigation of elastic-plastic wave parameters in iron and steel samples where explosive charges were detonated on their surfaces. A continuous recording device was used.

A small peak was obtained in the elastic precursor on the order of 0.1 μsec. It was also found that the precursor magnitude decreased in longer samples, or in other words, with distance. The magnitude of the precursor was rather arbitrarily defined as that of the pressure before

emergence of the plastic wave. That is, the pressure in the precursor may increase somewhat before the arrival of the shock. They explained the decay of the precursor as follows: "From the point of view of gas dynamics the phenomenon of decay is explained by a weak upward convexity of the elastic portion of the Hugoniot adiabat."

They also commented that the dynamic yield stress in iron and low-carbon steels is approximately equal to the force which locks dislocations. In alloyed steels the admixtures not only increase this locking force, but form additional obstacles to the motion of dislocations. The effect on the yield point is therefore less here than for low-alloy steels, since it takes longer for plastic flow to be initiated. In high-alloy steel, they found that there is a hardly noticeable strain-rate effect (i.e. increase in yield point).

Barker, Lundergan, and Herrmann (1964) carried out plate impact tests for 6061-T6 aluminum alloy. The aluminum was loaded up to 22 kb, and a slight strain-rate dependence was found. A method of charged probes was employed to measure the free surface velocity. A definite precursor was noted at the free surface.

The strain rate sensitivity was quite small. In general the dynamic stress-strain curve lay above the static, and there was a slight tailing off, probably as an effect of relaxation. They suggested that the dynamic curve fell from a curve of high strain-rate to one of lower strain-rate as the maximum stress was approached.

C. MICROSTRUCTURAL CHANGES ACCOMPANYING DYNAMIC LOADING

Only recently have microstructural changes accompanying plastic wave propagation been investigated. This has been done both to explain the effect of strain-rate behavior and the hardening of metals due to shocking.

Carrington and Gayler (1948) examined the microstructural changes accompanying the impact of projectiles. "The means by which the stress on impact was relieved depended on the material, and was first the formation of twins or 'compression bands,' i.e., by block movement of wedges of material within individual grains, or by cracking. When the applied stress could no longer be relieved in this way, plastic deformation occurred."

Cottrell (1957) discussed mechanisms for delayed yield; the predictions of this agreed very well with curves from Clark and Wood for delayed yielding. His calculations were based on the line energy of dislocations.

Dorn and Hauser (1962) showed that there may be both strain-rate dependent and strain-rate independent behavior of metals, depending upon the mechanism governing plastic deformation. When the mechanisms are thermally activated, the stress-plastic strain behavior will be strain-rate sensitive, and for athermal processes the curve will be strain-rate insensitive.

They used the intersection model to demonstrate strain-rate sensitivity when thermally activated mechanisms are predominant. It was pointed out that during a specific loading, different mechanisms may govern at different times. The authors felt that although there is a linear dependence of the strain rate on the stress, it is possible for the dislocations to acquire the velocity of sound, thus giving a limiting strain rate equal to the quantity nbc. They also discuss briefly temperature effects on strain rate.

Dorn and Rajnak (1963) summarized dislocation concepts of plastic-wave phenomena. They pointed out that dislocations reach maximum velocity in approximately 10^{-10} second, and because acceleration of dislocations can therefore be neglected, the empirical relationship

$$d\epsilon = h(\sigma, \epsilon)\, d\sigma + g(\sigma, \epsilon)\, dt$$

was suitable. Here h and g are functions to be determined by experiment.

Fugelso (1965) was one of the first to employ Mura's continuum theory of dislocations. He discussed the probability of the rate of dislocation's moving and obtained an average dislocation velocity. The strain-rate effect was considered to be the result of an increase in the number of dislocations, either by an increase by the Frank-Read source or by multiple cross-glide. It turned out that the equations of strain rate including the terms of dislocation velocity have the same form as used before (Dorn, Malvern, etc.) i.e., $g(\sigma, \epsilon)$ is a function of dislocation velocity, magnitude, and density.

He obtained curves for lithium flouride and a titanium alloy and compared these with his calculations. His results suggest further study of this approach.

Barker, Butcher, and Karnes (1965) employed a high resolution interferometer technique in the impacting of 1060 aluminum plates. They found an initial hump in the precursor wave travelling with elastic velocity and explained this in terms of dislocation motion and multiplication, as has been proposed by Johnston and others.

They used computer solutions for comparison with the experimental results. In these the plastic strain rate has the form:

$$\dot{\epsilon}^P = b \rho \bar{v}$$

where b is the magnitude of the Burger's vector, ρ is the mobile dislocation density, and $\bar{v}$ is the average velocity of the dislocations, which is a function of shear stress. They concluded, "In order for an overshoot in stress to develop in the wave shape, the stress in the material just behind the front must relax more rapidly than the stress at the wave front, i.e., the magnitude of the plastic strain rate for a given stress must be larger just behind the wave front than at the wave front. Otherwise, the entire elastic wave would decay at the same rate, and no overshoot would develop." They also noted that according to the expression above, the dislocation velocity must increase following passage of the wave front, and therefore the density of moving dislocations must increase.

They employed the equations for dislocation density and average velocity

$$\rho = \rho_o \left(1 + \frac{\alpha}{\rho_o} \epsilon^P\right)$$

$$\bar{v} = v_m e^{-B/\sigma}$$

where ρ_o is the original mobile dislocation density, v_m is the maximum attainable dislocation velocity, and σ is the shear stress. With this they obtained an initial sharp peak and then immediate relaxation. Their theory did not explain the spread of data which they obtained.

It should be noted here that this is a special case of the more general one discussed by Fugelso (1965), and they did not base their calculations entirely on the theory of dislocations as did Fugelso, i.e., their computations were still based on experimental data and a desire to match them with computer results.

D. MATHEMATICAL GENERALIZATIONS TO THREE DIMENSIONS

Apart from the experimental evidence of a loading rate effect, the concept of rate of strain vs stress has been approached from the point of view of viscosity in the fields of viscoelasticity and viscoplasticity, although they have not taken on such specific names until very recently. The theories developed for viscoplasticity have been applied to the strain-rate problem, since the effect is essentially one of viscosity during plastic flow.

Maxwell (1868) proposed the elastic-viscous solid in his paper "On the Dynamical Theory of Gases":

If the rate of strain is proportional to the stress, such a material is governed by the equation

$$\frac{d\sigma}{dt} = E\frac{d\epsilon}{dt} - \frac{\sigma}{T}$$

If the strain ϵ is constant, then one finds

$$\sigma = \sigma_1 e^{-t/T}$$

which indicates that the stress σ gradually disappears, or relaxes.

Bingham (1922) in his book Fluidity and Plasticity assumed that the material was characterized by a yield stress. When the stress in the material exceeded this yield stress, the material behaved as a perfectly viscous fluid. This can be written as

$$\tau = \tau_0 + \mu\gamma$$

where τ_0 is the yield stress (octahedral),
τ is the actual stress (octahedral),
μ is a constant of the material, and
γ is the octahedral rate of strain.

A body which behaves in this manner is now called a Bingham body.

Hohenemser and Prager (1932) generalized the concept of a Bingham body for three-dimensional behavior. In the case of pure shear, they assumed that the shear stress is some function of the shear strain alone, i.e.,

$$\tau = f(\gamma/2)$$

Every small distortional deformation can be composed of five pure shears, whose axes are sufficiently generally chosen:

$$e = \sum_{i=1}^{5} \gamma_i$$

The same holds for the corresponding stress deviators:

$$s = \sum_{i=1}^{5} \tau_i$$

This equation must be true for all shears which satisfy the preceding one, and besides it must reduce (in component form) to the first equation. This is seen to be the case only when:

$$s_{ij} = \frac{f(H)}{H} e_{ij}$$

where H is an invariant of e_{ij}. Then

$$Hs_{ij} = f(H) e_{ij}$$

if $s_{ij} = \begin{bmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and $e_{ij} = \begin{bmatrix} 0 & \gamma/2 & 0 \\ \sigma/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Then for distortional deformation, the first invariant is zero, and it has been shown elsewhere that plastic deformation depends primarily upon the second invariant. The only function of the second invariant which reduces to $\gamma/2$ is $I_2^{1/2}$, which is defined by

$$I_2 = \frac{1}{2} (e_{11}^2 + e_{22}^2 + e_{33}^2 + 2 e_{12}^2 + 2 e_{23}^2 + 2 e_{31}^2)$$

Then

$$I_2^{1/2} s_{ij} = f(I_2^{1/2}) e_{ij}$$

$$e_{ij} = \begin{bmatrix} 0 & \gamma/2 & 0 \\ \gamma/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} , \quad I_2 = \frac{1}{2} (2\frac{\gamma^2}{4}) = \frac{\gamma^2}{4}$$

Then

$$I_2^{1/2} = \gamma/2$$

and

$$\frac{\gamma}{2}\begin{bmatrix}0 & \tau & 0\\ \tau & 0 & 0\\ 0 & 0 & 0\end{bmatrix} = f(\frac{\gamma}{2})\begin{bmatrix}0 & \gamma/2 & 0\\ \gamma/2 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

or

$$\frac{\gamma}{2}\tau = f(\gamma/2)\,\gamma/2$$

$$\tau = f(\gamma/2)$$

as was desired. Assuming a simple functional relationship

$$\tau = c + \alpha\,\gamma/2$$

then we have, by substitution above,

$$s_{ij} = (c + \alpha\, I_2^{1/2})\,\frac{e_{ij}}{I_2^{1/2}}$$

or from

$$\tau - c = \alpha\,\gamma/2$$

we have

$$\frac{s_{ij}}{J_2^{1/2}}\,(J_2^{1/2} - c) = \alpha\, e_{ij}$$

where by definition

$$J_2 = s_{ij}\, s_{ij}$$

If τ depends on γ as in a Bingham body,

$$\tau - \tau_o = \mu\dot{\gamma}$$

then one finds (see Hohenemser and Proger, Equation 2', p. 224),

$$\frac{s_{ij}}{\sqrt{J_2}} (\sqrt{J_2} - \tau_o) = 2\,\mu e_{ij}$$

This is now the fundamental relationship between stress and strain rate for small deformations, if the body is of the Bingham type.

Prager (1937) introduced the term "tenseur d'exces". He related this to the state of stress in a material governed by some yielding surface. For a viscoplastic body, he assumed a linear relationship between the tenseur d'exces and the strain rate. Due to the isotropy of the solid, such a relation resembles that of a viscous fluid. He also assumes incompressibility for simplicity. The equations governing a viscoplastic body are then given by

$$\dot{\epsilon}_{ik} = 0 \qquad s < c,$$

$$\dot{\epsilon}_{ik} = \frac{s-c}{2\mu s} s_{ik} \qquad s \geq c$$

s being second invariant of stress, and the yield criterion being that of von Mises. This reduces for the special case of $c = 0$ to

$$s_{ik} = 2\mu\dot{\epsilon}_{ik}$$

which is the characteristic equation of an incompressible viscous fluid.

Ilyushin (1940) presented a very good monograph on viscoplastic materials.

Prager (1961) discussed a linearization in the theory of viscoplastic bodies to simplify the solution of boundary value problems. He rewrote the relation developed by Hohenemser and Prager in a form that is readily linearized:

$$F = (s_{pq}\, s_{qp})^{1/2} - k\sqrt{2}$$

He considered the constitutive equation

$$2\mu\dot{\varepsilon}_{ij} = \langle F \rangle \frac{\partial F}{\partial \sigma_{ij}} \tag{1}$$

where

$$\frac{\partial F}{\partial \sigma_{ij}} = \frac{s_{ij}}{(s_{pq}\, s_{qp})^{1/2}}$$

and $\langle F \rangle$ is defined as follows:

$$F = \begin{cases} 0 & F \leq 0 \\ F & F > 0 \end{cases}$$

For simple shear, Equation 1 reduces to

$$\mu\gamma = (\,|\tau| - k)\frac{\tau}{|\tau|}$$

which is the constitutive equation for a Bingham solid in simple shear. He concluded therefore that Equation 1 can be accepted as a suitable generalization of this constitutive equation for arbitrary types of stress and strain.

Perzyna (1963) rewrote the equations suggested by Prager (1961) in a slightly more general form:

$$\dot{e}_{ij} = \frac{1}{2\mu} s_{ij} + \gamma\Phi(F) \frac{s_{ij}}{J_2^{1/2}} \qquad J_2^{1/2} > k$$

$$\dot{e}_{ij} = \frac{1}{2\mu} s_{ij} \qquad J_2^{1/2} \leq k$$

$$\dot{\epsilon}_{ii} = \frac{1}{3B} \sigma_{ii}$$

where the function $\Phi(F)$ is to be based on experiment, k is yield stress in simple shear, and B the bulk modulus. F is defined as

$$F = J_2^{1/2}/k - 1$$

If we write

$$\dot{\epsilon}_{ij}^{P} = \gamma\Phi(F) \frac{s_{ij}}{J_2^{1/2}}$$

and then squaring both sides of this, we have

$$(I_2^{P})^{1/2} = \left(\frac{1}{2} \dot{\epsilon}_{ij}^{P} \dot{\epsilon}_{ij}^{P} \right)^{1/2} = \gamma\Phi(F)$$

We can then write

$$J_2^{1/2} = k\left[1 + \Phi^{-1}\left([I_2^{P}]^{1/2} / \gamma \right) \right]$$

where Φ^{-1} is the functional inverse of Φ. This is in effect a yield criterion, based on a strain-rate effect.

He then describes two special types of the function $\Phi(F)$:

$$\Phi(F) = F^{\delta} = \left(\frac{J_2^{1/2}}{k} - 1\right)^{\delta}$$

$$\Phi(F) = e^{F} - 1$$

The second, in a slightly different version, was that introduced by Malvern for one dimension

$$\dot{\varepsilon} = \frac{\dot{\sigma}}{E} + \gamma\left(e^{\sigma/\sigma_0 - 1} - 1\right)$$

Kaliski (1963) extended Perzyna's work to the work-hardening case, where the yield stress is no longer assumed constant. He wrote:

$$\dot{e}_{ij} = \frac{1}{2\mu}\dot{s}_{ij} + \varphi\Phi\left(\frac{J_2^{1/2}}{f\left(I_2^{1/2}\right)} - 1\right)\frac{s_{ij}}{J_2^{1/2}} \qquad J_2^{1/2} > f\left(I_2^{1/2}\right)$$

$$\dot{e}_{ij} = \frac{1}{2\mu}\dot{s}_{ij} \qquad J_2^{1/2} < f\, I_2^{1/2}$$

$$\dot{\varepsilon}_{ii} = \frac{1}{3B}\dot{\sigma}_{ii}$$

Naghdi and Murch (1963) proposed a general description of the mechanical behavior of viscoelastic/plastic solids, as follows:

> "It may be of interest to make a comparison with the work of Perzyna. Whereas in the present paper, after postulating the existence of a loading surface and deriving a loading criterion, we obtain (subsequent to the proof of convesity and normality) constitutive equations for viscoelastic/plastic solids. On the other hand, for elastic viscoplastic

solids, Perzyna begins by postulating both the existence of a 'static yield function' as well as the constitutive equations for which normality is assumed. Further, in [his paper] the stress is not required to satisfy a loading criterion and loading has the simple interpretation of occurring whenever the static yield condition is exceeded.

"It should be clear in view of considerable differences not only in the development of the present paper, but also in light of the different notions employed in constructing the present theory that the direct reduction to the constitutive equations of Perzyna is not possible except by rather artificial means. In fact ... Perzyna constitutive equations do not reduce to those in inviscid plasticity, since, as Perzyna himself notes, his yield function depends on the strain rate, whereas in inviscid plasticity the loading function in stress space depends on the plastic strain."

Perzyna (1964) also considered the case of work hardening. For a general case

$$F = F(\sigma_{ij}, \epsilon_{ij}^{P} = \frac{f\left(\sigma_{ij}, \epsilon_{ij}^{P}\right)}{\kappa} - 1$$

where

$$\kappa = \kappa(W_p) = \kappa\left(\int_0^{\epsilon_{ij}^{P}} \sigma_{ij}\, d\epsilon_{ij}^{P}\right)$$

is a work-hardening parameter. Also

$$f(\sigma_{ij}, \epsilon_{ij}) = \varkappa(W_p)\left\{1+\Phi^{-1}\left[\frac{I_2^{P^{1/2}}}{\gamma}\left(\frac{1}{2}\frac{\partial f}{\partial \sigma_{k\ell}}\frac{\partial f}{\partial \sigma_{k\ell}}\right)^{-1/2}\right]\right\}$$

As a special case, let $f(\sigma_{ij}) = (J_2)^{1/2}$. Then

$$\dot{e}_{ij} = \frac{1}{2\mu}\dot{s}_{ij} + \gamma\left(\Phi\left(\frac{\sqrt{J_2}}{\varkappa} - 1\right)\right)\frac{s_{ij}}{\sqrt{J_2}} \qquad \dot{\epsilon}_{ii} = \frac{1}{3B}\dot{\sigma}_{ii}$$

and according to the above equation, the dynamical yield criterion has the form

$$\sqrt{J_2} = \varkappa(W_p)\left[1 + \Phi^{-1}\left(\frac{\sqrt{I_2^P}}{\gamma}\right)\right]$$

For one-dimensional states the above relation yields:

$$\dot{\epsilon} = \frac{\dot{\sigma}}{E} + \gamma^* \left\langle \Phi\left[\frac{\sigma}{\varphi(\epsilon^P)} - 1\right]\right\rangle$$

where $\gamma^* = (2/3)\,\gamma$

$\varphi(\epsilon^P) = \sqrt{3}\,\varkappa(W_p)$

This was first introduced by Malvern.

If we assume now

$$F = \frac{\sqrt{J_2}}{k} - 1$$

the dynamic yield criterion becomes

$$\sqrt{J_2} = k\left[1 + \Phi^{-1}\left(\frac{\sqrt{I_2^P}}{\gamma}\right)\right]$$

Perzyna (1964), in another article, selected different types of the function $\Phi(F)$. These were of the forms:

$$\Phi(F) = F^{\delta}, \text{ or } F, \text{ or } e^{F} - 1$$

and

$$\Phi(F) = \sum_{\alpha=1}^{N} A_{\alpha}\left[e^{F^{\alpha}} - 1\right] \qquad \text{or} \qquad = \sum_{\alpha=1}^{N} B_{\alpha} F^{\alpha}$$

He proceeded then to compare these with experiments, but did not find any particularly good fits for any of them. In comparing with data of Hauser, et al, for example, he compared his fit only with the lower strain rates, and left off the higher one, where the fit had become progressively worse.

SECTION III
APPLICATIONS

It was decided that the most profitable geometry for studying strain-rate effects was that of uni-axial strain, both because the geometry is simple, and because the computer time for a one-dimensional configuration is much less than for a two-dimensional study. The Physics International "POD" code was employed. The exponential form of Perzyna's generalization was chosen (in one-dimensional stress):

$$\dot{\epsilon} = \frac{\dot{\sigma}}{E} + \gamma^*(e^{(\frac{\sigma}{\varphi} - 1)} - 1)$$

or

$$\sigma = \varphi(\epsilon)\left[1 + \ln(1 + \frac{\dot{\epsilon}^P}{\gamma^*})\right]$$

where

$$\gamma^* = \frac{\dot{\epsilon}^P}{\left[e^{(\frac{\sigma}{\varphi} - 1)} - 1\right]}$$

This was fitted to curves from Dorn, as a check on the theory.

For the case of uni-axial strain, however, a more general yield criterion in line with those incorporated in the POD code was needed. From Perzyna we have

$$\sqrt{J_2} = \varkappa\left[1 + \ln\left(1 + \frac{\sqrt{I_2^P}}{\gamma}\right)\right]$$

But for uni-axial stress we have

$$I_2^P = \frac{1}{2}\left(e_1^2 + e_3^2 + e_3^2\right) = \frac{1}{2}\left[\left(\frac{2}{3}\,\dot{\epsilon}\right)^2 + 2\left(\frac{1}{3}\,\dot{\epsilon}\right)^2\right] = \frac{\dot{\epsilon}^2}{3}$$

Therefore,

$$\sqrt{I_2^P} = \frac{\dot{\epsilon}}{\sqrt{3}}$$

Therefore, in conjunction with POD and the von Mises yield criterion used therein, we have

$$s_1^2 + s_2^2 + s_3^2 = 2\chi^2\left[1 + \ell n\left(1 + \frac{\dot{\epsilon}}{\gamma\sqrt{3}}\right)\right]^2$$

This expression served as the basis for the preliminary one-dimensional computer runs, POD 84 and POD 85, the second being the same configuration with finer zones in order to check the sensitivity of the velocities and the strain rates.

After some corrections were made, it was decided to run a similar problem, only with strain-hardening omitted for simplicity. Here three POD runs (93, 94, 95) were carried out, the first with strain-rate independent yield, the second with the yield stress defined by

$$Y = Y_o\left[1 + \ell n\left(1 + \frac{|\dot{\epsilon}_r|}{0.002125}\right)\right]$$

and the third the same as the second with the time step reduced by one-half, again as a check for sensitivity. Here POD 93 ran well, giving precisely the results expected from strain-rate independent theory. POD 94 showed no yielding, and POD 95 was little different from POD 94. Absence of yielding resulted from oscillations in computed particle velocity and strain rate, which were sufficient to give a very large strain rate at all times, even after passage of the wave front, when the strain rate is known to diminish rapidly.

An attempt to damp this oscillation was made by the introduction of a artificial linear viscosity Q in POD 96 on the same configuration. There was no noticeable improvement.

An integral formulation of the strain-rate effect was then used as a means of summing the oscillations and thereby smoothing them. We will only outline the details here.

It was decided to use a linear rate dependence in order to deal with a simple linear differential equation (in one-dimensional stress)

$$E_o \frac{d\epsilon}{dt} = \frac{d\sigma}{dt} + k(\sigma - \sigma_o)$$

This yields the solution

$$\sigma(t) = \int_{-\infty}^{t} e^{-k(t-\tau)} \left[E_o \dot{\epsilon} + k\sigma_o \right] d\tau$$

Put into a finite difference form, where the yield stress at time step n + 1 depends upon the yield stress at the previous step n, the equation becomes

$$\sigma^{n+1} = \left[E_o \dot{\epsilon}^{n+1} + k\sigma_o \right] \Delta t^{n+1/2} + e^{-k\Delta t^{n+1/2}} \sigma^n$$

We wish to extend this now to one-dimensional strain. Beginning with the expression

$$\dot{e}_{ij} = \frac{1}{2\mu} s_{ij} + \gamma \left(\frac{\sqrt{J_2}}{\kappa} - 1 \right) \frac{s_{ij}}{\sqrt{J_2}}$$

we arrive at

$$\frac{ds_x}{dt} + \frac{3}{2} \beta s_x = 2\mu \frac{de_x}{dt} + \beta\sigma_o$$

where σ_o is the yield stress in a tension test, and

$$\beta = \frac{4\mu\gamma}{\sqrt{3}\sigma_o}$$

in $\sec^{-1}$. Comparing this with the one-dimensional stress case, we can write the solution as

$$s_x(t) = \int_{-\infty}^{t} e^{-3/2\beta(t-\tau)} \left[2\mu\dot{e}_x + \beta\sigma_o \right] d\tau$$

From one-dimensional stress we find

$$\gamma = \frac{\sqrt{3 k\sigma_o}}{2E}$$

and therefore

$$\beta = \frac{2\mu}{E} k$$

where k is the proportionality constant for the linear strain-rate dependence.

In one-dimensional strain, when the material has reached yield,

$$\bar{s}_x = \frac{2}{3} Y$$

Now, as in the one-dimensional stress case, we can write

$$s_x^{n+1} = 2\mu \left[\dot{e}_x^{n+1} + k \frac{\sigma_o}{E} \right] \Delta t^{n+1/2} + e^{-\frac{3\mu}{E} k\Delta t^{n+1/2}} s_x^n$$

where s_x is to be replaced by $\bar{s}_x$ for a yield criterion.

If we define $\sigma_o = Y(0)$ and $|\dot{e}| = |\dot{\epsilon} - \frac{1}{3}\frac{\dot{V}}{V}|$, we find

$$Y^{n+1} = \frac{3}{2}\left\{ 2\mu\left[\left|\dot{\epsilon}^{n+1/2} - \frac{1}{3}\frac{\dot{V}}{V}^{n+1/2}\right| + k\frac{\sigma_o}{E}\right]\Delta t^{n+1/2} + \frac{2}{3}\, e^{-\left(\frac{3\mu}{E}k\Delta t^{n+1/2}\right)} Y^n \right\}$$

This yield criterion provided the basis for POD 97, where the constant k of proportionality was chosen as 10^6/sec or 1/μsec. This was chosen from Malvern (1951). Unfortunately, because of the oscillation already mentioned, the computed yield stress was still much too high.

Another trial employed a curve smoothing method on the particle velocity—versus—distance curve, to try to eliminate the oscillations in strain rate. This was done with a five-point procedure in POD 103A. No improvement was noted.

It was then decided that the zero strain rate could be accomplished by eliminating the specific rate-dependent yield stress after passage of the front and using instead an enforced relaxation time. The imposed yield stress then had the form $Y = Y_o$ until the strain rate exceeded 10^{-5}, and then it was

$$Y^{n_o+n} = 3\mu\left[\left|\frac{2}{3}\epsilon^{n_o+n}\right| + \frac{Y_o}{E}\right]\Delta t^{n_o+n} + Y^{n_o+n-1}$$

until the strain rate changed sign after n cycles. This was then called the dynamic yield stress $Y_D = Y^{n_o+n}$. Thereafter, with $Y_D - Y_o = Y_{ex}$,

$$Y = Y_o - Y_{ex} d^{-t/\tau}$$

where

$$\tau = \frac{70}{Y_D - Y_o}$$

where the value for relaxation time was arbitrarily selected for this exploratory investigation. Here a definite decay in the precursor was noted, but since the explosive loading was used the relaxation phenomenon was difficult to study.

A final POD run was made (POD 126) where the similar type of rate-dependent yield stress and relaxation time was used. However, a maximum dynamic yield stress of three times the static one was arbitrarily enforced and a constant relaxation time was used. The results of this run were ambiguous enough so that the POD project was interrupted pending analysis.

In the course of successive simplifications of an intractable computer problem, this last run had brought us back to the starting point, i.e., use of a constant dynamic yield stress. If the rate is such that the maximum yield stress specified is reached and the time duration of the run is short enough, it is seen that this is equivalent to a constant dynamic yield stress. Longer times, however, would give different results.

All of the above computer runs were based on empirical formulae, which used output of the computer for time n to perform further computations for time n+1. In the program used, oscillations due to amplification of round-off errors caused excessive difficulty. In the case where dislocation motion is used as the basis for strain-rate effects, less oscillatory parameters (such as stress) are involved in the computations, and the results would be expected to be more realistic. It would seem then that the approaches taken by Fugelso (1965) and Barker, et al (1965) might be the most fruitful for further computational studies of strain-rate effects in metals.

SECTION IV

DISCUSSION AND CONCLUSIONS

It appears that the rate of loading or energy input into a material determines the magnitude of the dynamic yield stress, at least up to some limit. One should be careful in using the term "yield stress" in a dynamic situation, because no material can ever yield instantaneously. As soon as some stress is developed in the material, dislocations begin to move, causing micro-yielding. A finite time is required for these to combine and furnish macro-yielding. Such integration of motion has been proposed by Cottrell (1957). Just how high the initial elastic stress in a precursor will be and why there may be an upper limit to this is not clear. It has been noted (Ivanov, et al, 1963) that the lower the ratio of the static yield system to the ultimate strength, the higher the dynamic stress. A material which is well worked and has a very high static yield stress will therefore exhibit only a very small increase in yield stress under dynamic loading.

With a stress higher than the static yield stress, a precursor propagates through the material. Behind the front the material suddenly finds itself at a stress level higher than the static yield stress, and dislocations begin to move. Some experiments indicate that there is an actual delay time before the onset of yielding. Mathematically, if the yield stress were to relax rather than the actual stress, the phenomenon could be accounted for, since there would then be no yielding until the yield stress dropped below the actual stress. Clearly, the concept of yield stress is highly artificial in this case. It would be more proper to speak in terms of moving dislocations. Some authors have introduced relaxation into the actual stresses in place of the yield stress, but it is not clear what implications this may have on the behavior. With such a description, one does not obtain a delay time, but an actual relaxation of the stresses.

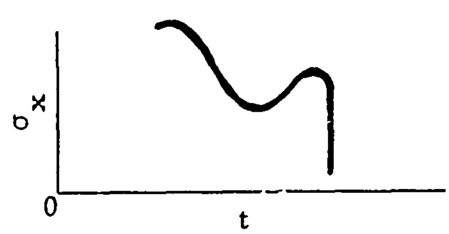
The concept of strain rate is confusing, because the strain rate at a shock front is essentially infinite. Moreover, the strain rate behind a strong shock can be zero. The recent studies of Fugelso and Barker, et al, are based upon the motion of dislocations. The plastic strain rate is proportional to the quantity nbv, where n is the density of moving dislocations, b is the magnitude of the Burger's vector, and v is the average velocity of moving dislocations. It appears likely that the density of dislocations must increase in order to obtain an increase in yielding behind a precursor traveling with the elastic velocity. This density increase would then cause an increase in plastic flow and, therefore, a relaxation of stress behind the wave front. The dislocation velocity is usually assumed to be some monotonically increasing function of stress up to some limiting velocity.

The dislocation model described above suggests the following qualitative description of a partially developed stress wave. An elastic wave arrives, placing the material in a stress which is higher than it can withstand statically without yielding. It begins to yield, i.e., dislocations begin to move along glide planes, causing irreversible strains in the material. As dislocations build up, the plastic strain rate increases, and the stress behind the shock front drops even faster than the stress at the shock front. If the elastic wave were allowed to proceed far enough, elastic wave amplitude would approach the static yield stress. The dislocation velocity would become small and the plastic strain rate would approach zero.

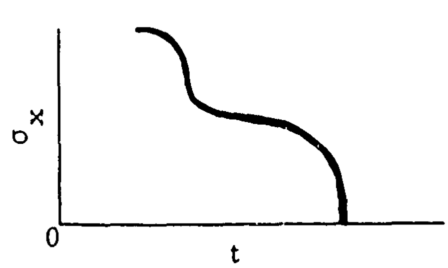
In general, when the plastic wave arrives, the material is still yielding. According to the above model, the plastic strain rate will increase if the dislocation velocity and/or density are increased. The dislocation density is not likely to increase instantaneously. Therefore the increase in plastic strain rate is governed by the dislocation velocity. If this velocity is already close to its upper limit, the increase in plastic strain will be small.

If the material in question is hardenable, such as aluminum, the "yield stress" is increased as the material flows. If the yield stress increases with strain faster than the stress relaxation, then no peak or hump in the precursor will be noted. This is observed in hardenable steels and aluminum (for example, Jones, et al, 1962). Iron and other non-hardenable steels, on the other hand, do in fact exhibit a definite peak. Barker, et al (1965) noticed such a peak in pure aluminum.

The proposed behavior suggests the following qualitative curves. If the material is not hardenable, a peak will occur in the precursor and the stress-time curve takes the following form:



If the material is hardenable, and hardens as fast or faster than the stress relaxes, the curve will take the following shape:



A sharp discrepancy between strain-rate-effect characterizations examined in this report should be noted. If dislocation motion is used to explain the rate effect (for example, Barker, et al, 1965), the plastic strain rate has the form

$$\dot{\epsilon} = \frac{\dot{\sigma}}{E} + \dot{\epsilon}^P$$

and

$$\dot{\epsilon}^P = b\rho_o\left(1 + \frac{\alpha}{\rho_o}\,\epsilon^P\right)V_m e^{-B/\sigma}$$

For $\alpha = 0$ we have

$$\sigma = -\frac{B}{\ln\left(\dot{\epsilon}^P / b\rho_o V_m\right)}$$

If, on the other hand, one assumes a law of the form

$$\dot{\epsilon} = \frac{\dot{\sigma}}{E} + \frac{g(\sigma, E)}{E}$$

and

$$\dot{\epsilon}^P = k\left[e^{(\sigma/\sigma_o - 1)} - 1\right]$$

the yield stress has the following form (Malvern type):

$$\sigma = \sigma_o\left[1 + \ln\left(\frac{\dot{\epsilon}^P}{k} + 1\right)\right]$$

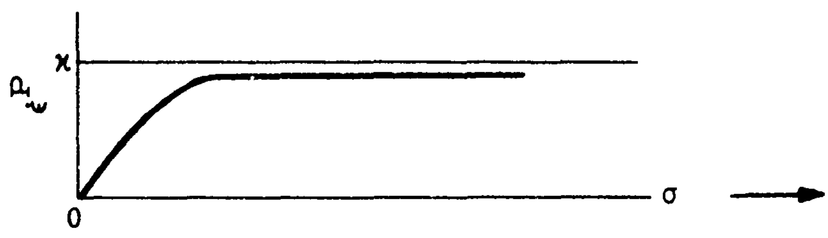
Now the former characterization has the form

$$\dot{\epsilon}^P = b\rho_o V e^{-B/\sigma} = \chi e^{-B/\sigma}$$

and the latter the form

$$\dot{\epsilon}^P = k\left[e^{(\sigma/\sigma_o - 1)} - 1\right]$$

If one plots these two curves, the following schematic curves are obtained. For the case of dislocation motion:



For the case of exponential overstress:

It is seen that these are intrinsically inconsistent types of behavior. The first yields a limiting plastic strain rate, as was suggested by Dorn. Since the second does not, its applicability appears impaired. The second type of fit has been commonly used and correlated with experiments, even though it clashes with the more feasible explanation of the behavior of the material. Its accuracy as an approximation is clearly limited to relatively low strain rates.

Both Fugelso and Barker, et al, have adapted a dislocation theory model similar to that described above to a finite difference computer program for calculating the propagation of stress waves. Both have reported some success.

We conclude that the most promising approach to the calculation of strain-rate effects in stress wave propagation appears to be a model based upon dislocation theory.

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13 ABSTRACT

A thorough summary of the literature treating strain-rate behavior in metals, 1948-1965, is given. Attempts to apply these theories and results to specific problems to be solved on a computer are discussed. A proposed qualitative description of the behavior of metals under dynamic loading is also presented.

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